Measurement

Contents:

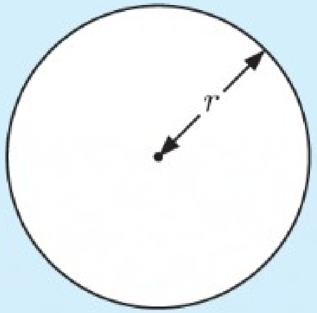
A Circles, arcs, and sectors
B Surface area
C Volume
D Capacity

A

CIRCLES, ARCS, AND SECTORS

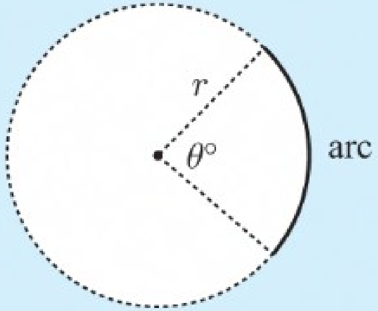
For a **circle** with radius r:

- the **circumference** $C = 2\pi r$
- the **area** $A = \pi r^2$.



An **arc** is a part of a circle which joins any two different points. It can be measured using the angle $\theta°$ subtended by the points at the centre.

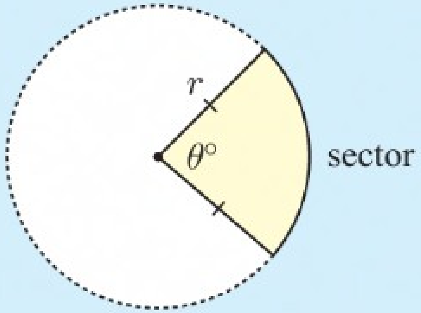
$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$



A **sector** is the region between two radii of a circle and the arc between them.

$$\begin{aligned}\text{Perimeter} &= \text{two radii} + \text{arc length} \\ &= 2r + \frac{\theta}{360} \times 2\pi r\end{aligned}$$

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$



Example 1

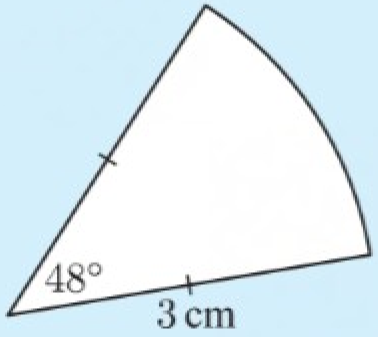
Self Tutor

For the given figure, find to 3 significant figures:

a the length of the arc

b the perimeter of the sector

c the area of the sector.



a $$\text{Arc length} = \frac{\theta}{360} \times 2\pi r = \tfrac{48}{360} \times 2\pi \times 3 \text{ cm} \approx 2.51 \text{ cm}$$

b $$\text{Perimeter} = 2r + \text{arc length} \approx 2 \times 3 + 2.51 \text{ cm} \approx 8.51 \text{ cm}$$

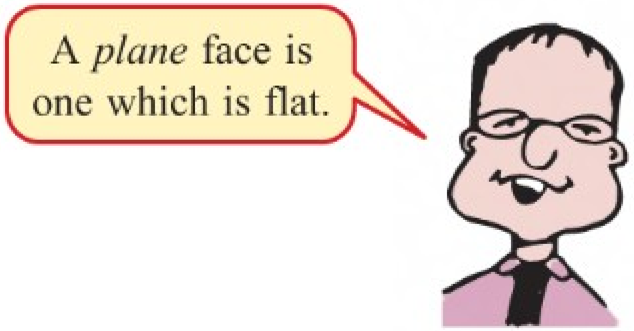
c $$\text{Area} = \frac{\theta}{360} \times \pi r^2 = \tfrac{48}{360} \times \pi \times 3^2 \approx 3.77 \text{ cm}^2$$

B SURFACE AREA

SOLIDS WITH PLANE FACES

> The **surface area** of a three-dimensional figure with plane faces is the sum of the areas of the faces.

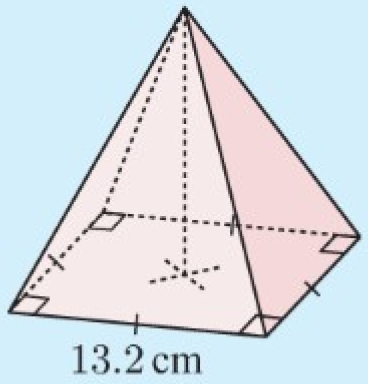
The surface area is therefore the same as the area of the **net** required to make the figure.


A *plane* face is one which is flat.

Example 2

Self Tutor

The pyramid shown is 10.8 cm high.
Find its surface area.



The net of the pyramid includes one square with side length 13.2 cm, and four isosceles triangles with base 13.2 cm.

Let the height of the triangles be h cm.

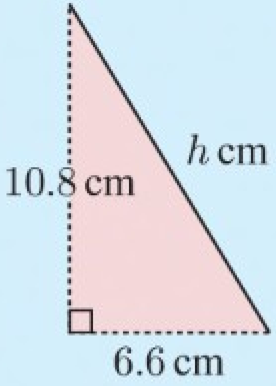
Now $h^2 = 10.8^2 + 6.6^2$ {Pythagoras}

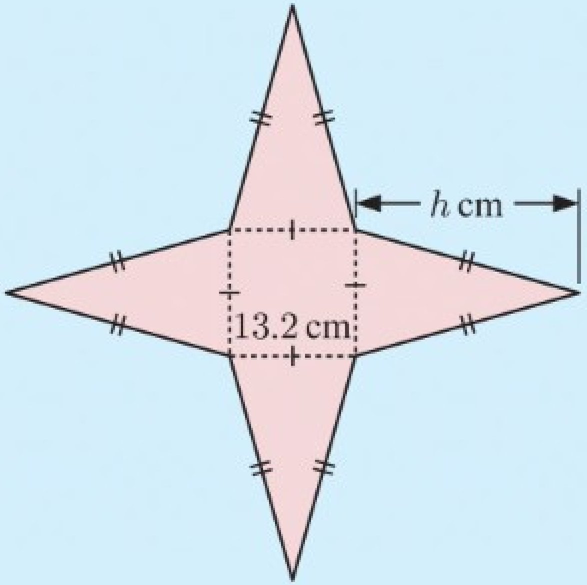
$\therefore$ $h = \sqrt{10.8^2 + 6.6^2} \approx 12.66$

$\therefore$ the surface area

$\approx 13.2^2 + 4 \times (\frac{1}{2} \times 13.2 \times 12.66)$ cm^2

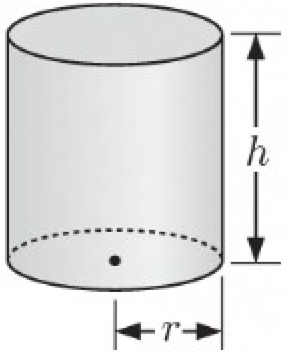
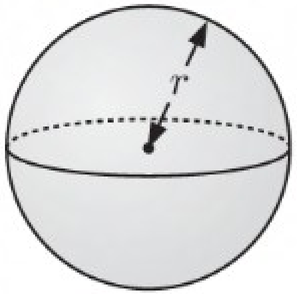
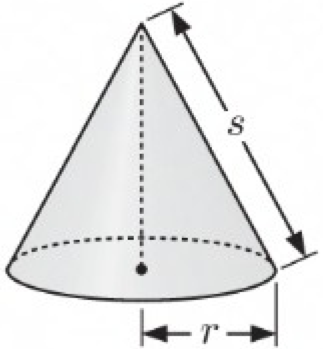
≈ 508 cm^2





SOLIDS WITH CURVED SURFACES

These objects have curved surfaces, but their surface areas can still be calculated using formulae.

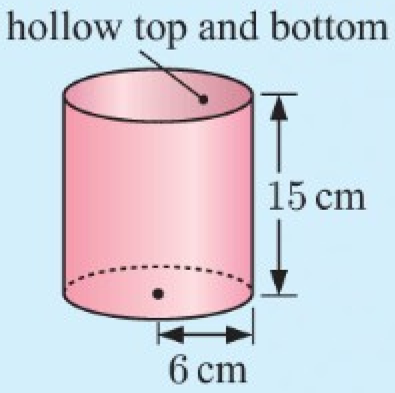
Cylinder	**Sphere**	**Cone**
		
$A =$ **curved surface** **+ 2 circular ends** $= 2\pi rh + 2\pi r^2$	$A = 4\pi r^2$	$A =$ **curved surface** **+ circular base** $= \pi rs + \pi r^2$

Example 3

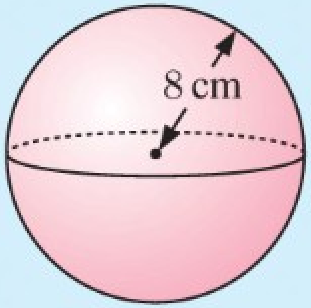
Self Tutor

Find, to 1 decimal place, the outer surface area of:

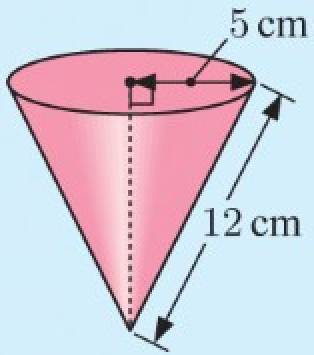
a



b



c



a The cylinder is hollow top and bottom, so we only have the curved surface.

$$A = 2\pi r h$$
$$= 2 \times \pi \times 6 \times 15$$
$$\approx 565.5 \text{ cm}^2$$

b
$$A = 4\pi r^2$$
$$= 4 \times \pi \times 8^2$$
$$\approx 804.2 \text{ cm}^2$$

c
$$A = \pi r s + \pi r^2$$
$$= \pi \times 5 \times 12 + \pi \times 5^2$$
$$\approx 267.0 \text{ cm}^2$$

Example 4 **Self Tutor**

The length of a hollow pipe is three times its radius.

a Write an expression for its outer surface area in terms of its radius r.

b If the outer surface area is 301.6 m^2, find the radius of the pipe.

---

a Let the radius be r m, so the length is $3r$ m.

$$\begin{aligned}\text{Surface area} &= 2\pi r h\\ &= 2\pi r \times 3r\\ &= 6\pi r^2\text{ m}^2\end{aligned}$$

b The surface area is 301.6 m^2

$$\begin{aligned}\therefore\ 6\pi r^2 &= 301.6\\ \therefore\ r^2 &= \frac{301.6}{6\pi}\\ \therefore\ r &= \sqrt{\frac{301.6}{6\pi}} \qquad \{\text{as } r > 0\}\\ \therefore\ r &\approx 4.00\end{aligned}$$

The radius of the pipe is 4 m.

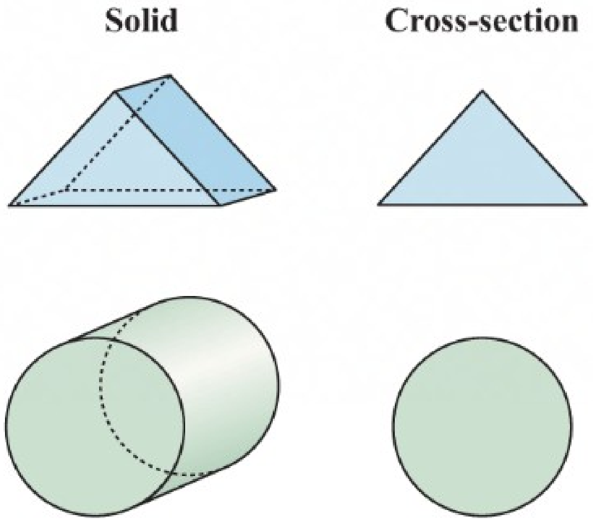
C VOLUME

The **volume** of a solid is the amount of space it occupies.

SOLIDS OF UNIFORM CROSS-SECTION

In the triangular prism alongside, any vertical slice parallel to the front triangular face will be the same size and shape as that face. Solids like this are called *solids of uniform cross-section.* The cross-section in this case is a triangle.

Another example is a cylinder which has a circular cross-section.

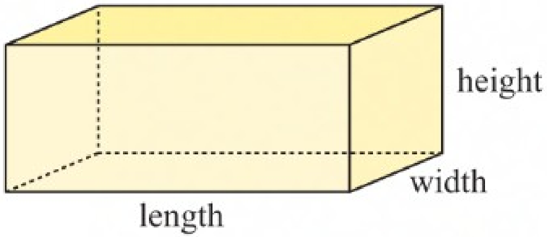


For any solid of uniform cross-section:

Volume = area of cross-section × length

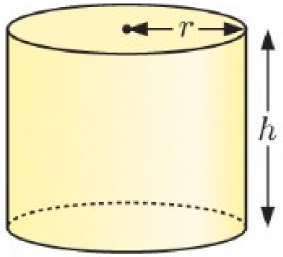
In particular, we can define formulae for the volume of:

- rectangular prisms



Volume = length × width × height

- cylinders



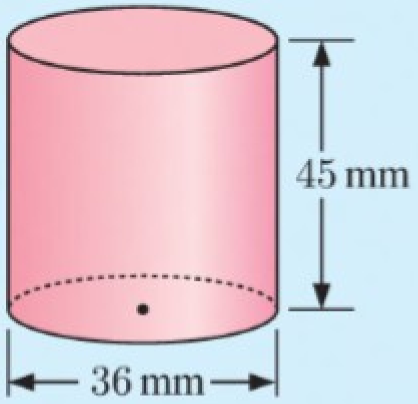
Volume $= \pi r^2 h$

Example 5

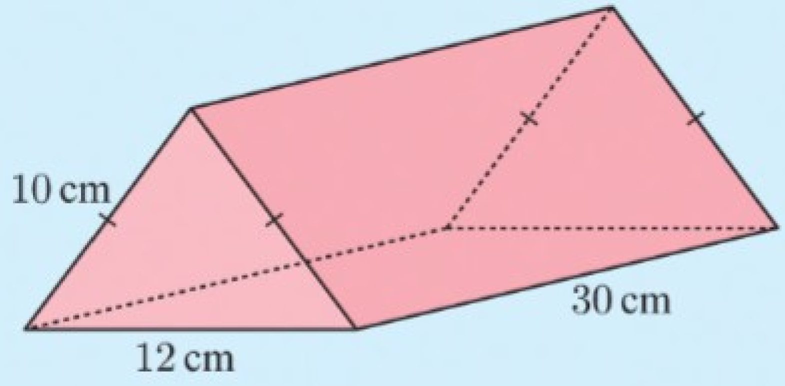
Self Tutor

Find the volume of:

a



b

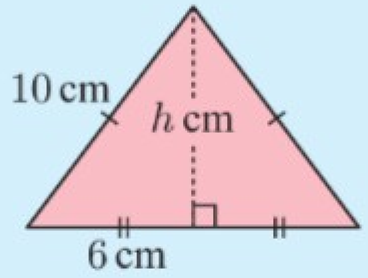


a $V = \pi r^2 h$

$= \pi \times 18^2 \times 45 \text{ mm}^3$

$\approx 45\,800 \text{ mm}^3$

b



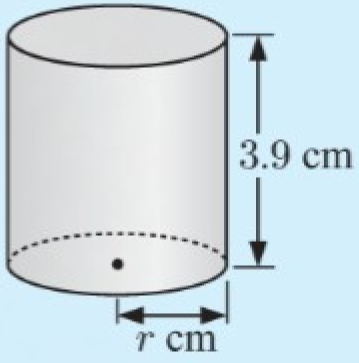
Let the prism have height h cm.

$$\begin{aligned} h^2 + 6^2 &= 10^2 \quad \{\text{Pythagoras}\} \\ \therefore\ h^2 + 36 &= 100 \\ \therefore\ h^2 &= 64 \\ \therefore\ h &= 8 \quad \{\text{as } h > 0\} \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \text{area of cross-section} \times \text{length} \\ &= (\tfrac{1}{2} \times 12 \times 8) \times 30 \text{ cm}^3 \\ &= 1440 \text{ cm}^3 \end{aligned}$$

Example 6 Self Tutor

Find, to 3 significant figures, the radius of a cylinder with height 3.9 cm and volume 54.03 cm^3.



$$V = 54.03 \text{ cm}^3$$

$$\therefore \quad \pi \times r^2 \times 3.9 = 54.03 \qquad \{V = \text{area of cross-section} \times \text{height}\}$$

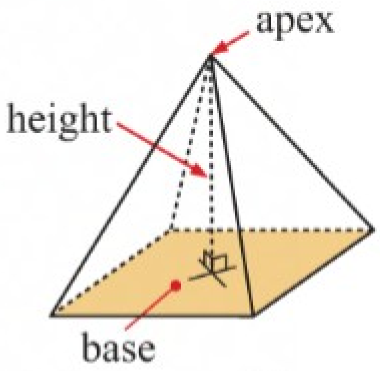
$$\therefore \quad r^2 = \frac{54.03}{\pi \times 3.9} \qquad \{\text{dividing both sides by } \pi \times 3.9\}$$

$$\therefore \quad r = \sqrt{\frac{54.03}{\pi \times 3.9}} \approx 2.10 \qquad \{\text{as } r > 0\}$$

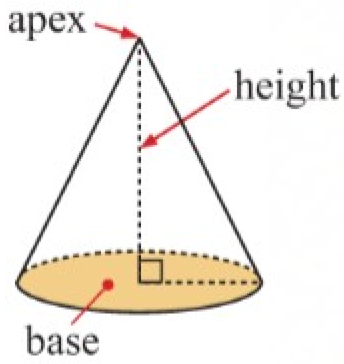
The radius is approximately 2.10 cm.

TAPERED SOLIDS

For pyramids and cones, the cross-section is not uniform. Rather, the cross-sections are a set of similar shapes which get smaller and smaller as we approach the apex. We call these **tapered solids**.



pyramid



cone

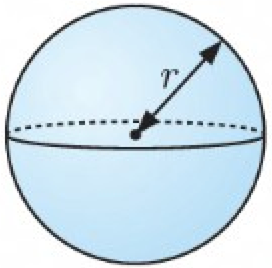
From the **Investigation**, you should have established that the volume of any tapered solid is given by:

$$\textbf{Volume} = \tfrac{1}{3}(\textbf{area of base} \times \textbf{height})$$

It is a third of the volume of the solid of uniform cross-section with the same base and height.

SPHERES

In previous years you should have seen that the volume of a sphere with radius r is given by



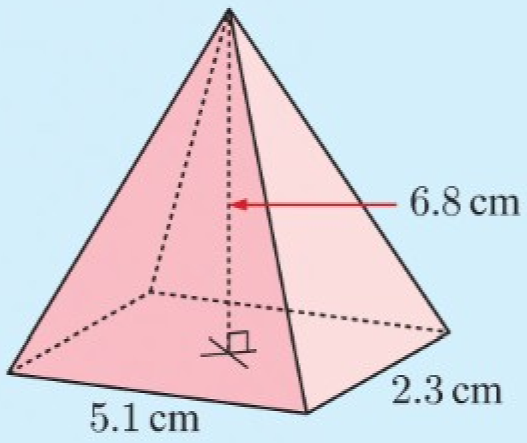
$$\textbf{Volume} = \tfrac{4}{3}\pi r^3$$

Example 7

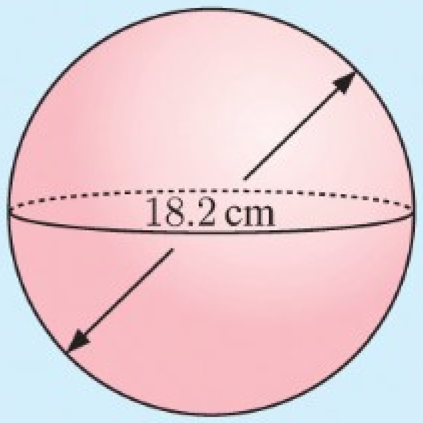
🔊 **Self Tutor**

Find the volume of each solid:

a



b



a $V = \frac{1}{3}(\text{area of base} \times \text{height})$

$= \frac{1}{3}(\text{length} \times \text{width} \times \text{height})$

$= \frac{1}{3}(5.1 \times 2.3 \times 6.8)\ \text{cm}^3$

$\approx 26.6\ \text{cm}^3$

b $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\frac{18.2}{2}\right)^3$

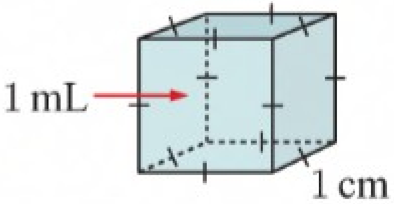
$\approx 3160\ \text{cm}^3$

D CAPACITY

The **capacity** of a container is the quantity of fluid it is capable of holding.

Notice that the term "capacity" belongs to the container rather than the fluid itself. The capacity of the container tells us what *volume* of fluid fits inside it. The units of volume and capacity are therefore linked:

1 mL of water occupies 1 cm^3 of space.



Volume		*Capacity*
1 cm^3	$\equiv$	1 mL
1000 cm^3	$\equiv$	1 L
1 m^3	$\equiv$	1 kL
1 m^3	$\equiv$	1000 L

$\equiv$ means "is equivalent to".

Example 8

Self Tutor

Find the volume of liquid which will fit in a container with capacity:

a 9.6 L

b 3240 L

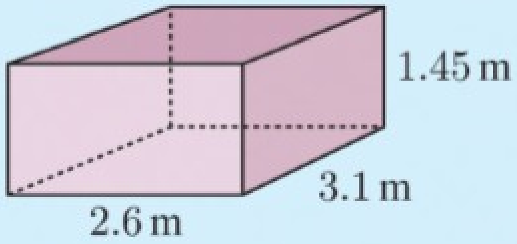
a 9.6 L
$= (9.6 \times 1000)\ \text{cm}^3$
$= 9600\ \text{cm}^3$

b 3240 L
$= (3240 \div 1000)\ \text{m}^3$
$= 3.24\ \text{m}^3$

Example 9

Self Tutor

Find the capacity of a 2.6 m by 3.1 m by 1.45 m tank.



$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 2.6 \times 3.1 \times 1.45 \text{ m}^3 \\ &= 11.687 \text{ m}^3 \end{aligned}$$

The tank's capacity is 11.687 kL.

Example 10 **Self Tutor**

17.3 mm of rain falls on a flat rectangular shed roof which has length 10 m and width 6.5 m. All of the water goes into a cylindrical tank with base diameter 4 m. By how many millimetres does the water level in the tank rise?

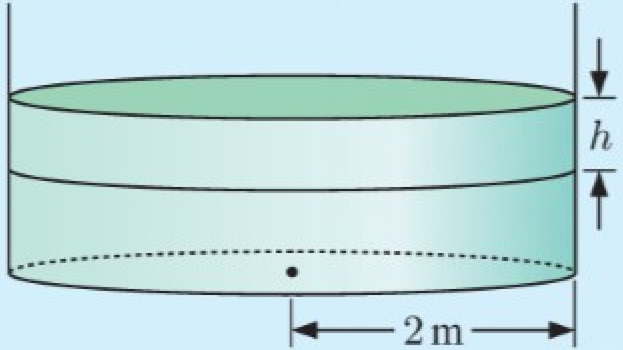
For the roof: The dimensions of the roof are in m, so we convert 17.3 mm to metres.

$$17.3 \text{ mm} = (17.3 \div 1000) \text{ m} = 0.0173 \text{ m}$$

$$\begin{aligned}\text{The volume of water collected by the roof} &= \text{area of roof} \times \text{depth}\\ &= 10 \times 6.5 \times 0.0173 \text{ m}^3\\ &= 1.1245 \text{ m}^3\end{aligned}$$

For the tank: The volume added to the tank

$$\begin{aligned}&= \text{area of base} \times \text{height}\\ &= \pi \times 2^2 \times h \text{ m}^3 = 4\pi \times h \text{ m}^3\end{aligned}$$



The volume added to the tank must equal the volume which falls on the roof, so

$$4\pi \times h = 1.1245$$

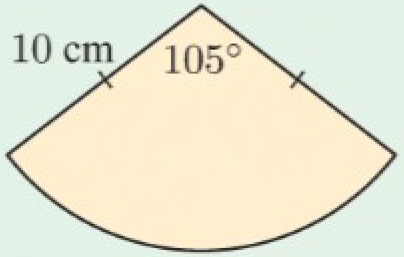
$$\therefore \quad h = \frac{1.1245}{4\pi} \qquad \{\text{dividing both sides by } 4\pi\}$$

$$\therefore \quad h \approx 0.0895 \text{ m}$$

$\therefore$ the water level rises by about 89.5 mm.

REVIEW SET 6A

1 For the given sector, find to 3 significant figures:

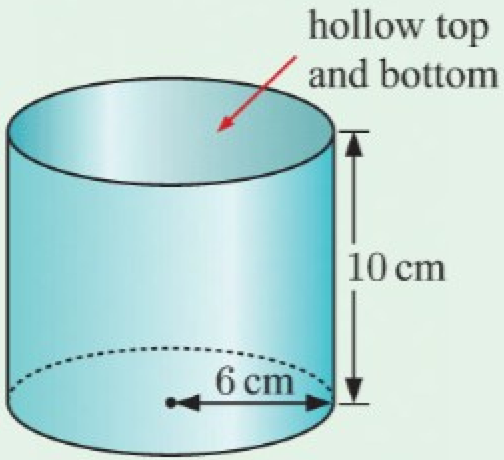


- **a** the length of the arc
- **b** the perimeter of the sector
- **c** the area of the sector.

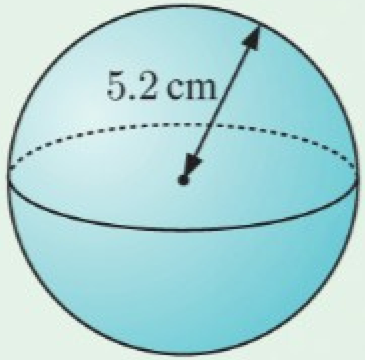
2 Find the radius of a sector with angle 80° and area 24π cm^2.

3 Find, to 1 decimal place, the outer surface area of:

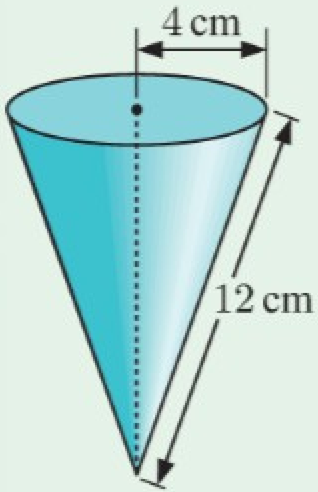
a



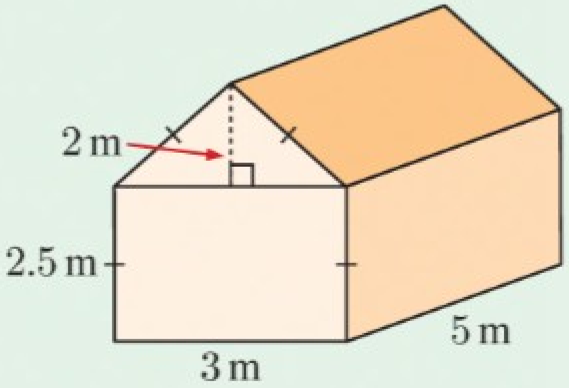
b



c



4 A tool shed with the dimensions illustrated is to be painted with two coats of zinc-alum. Each litre of zinc-alum covers 5 m^2 and costs $8.25. It must be purchased in whole litres.

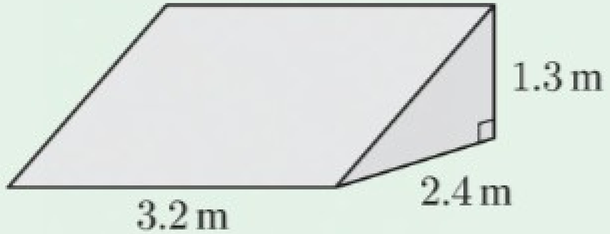


- **a** Find the area to be painted, including the roof.
- **b** Find the total cost of the zinc-alum.

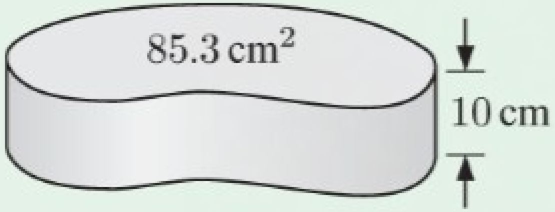
5 A fish farm has *six* netted cylindrical cages open at the top. The cylinders have depth 17 m and diameter 7.5 m. Find the total area of netting in the cages.

6 Calculate, to 3 significant figures, the volume of:

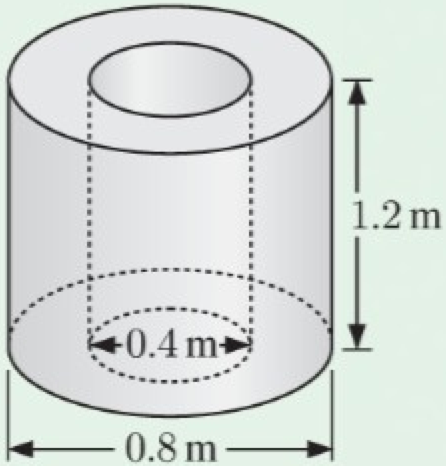
a



b



c



7 Tom has just had a load of sand delivered. The sand is piled in a cone with radius 1.6 m and height 1.2 m. Find the volume of the sand.

8 A manufacturer of spikes has 245 L of molten iron. If each spike contains 15 mL of iron, how many spikes can be made?

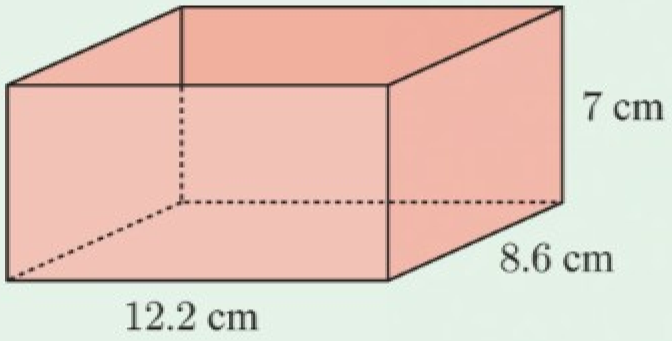
9 A plastic beach ball has radius 27 cm. Find its volume.

10 The capacity of a petrol tank is 65 L. State the volume of petrol required to fill the tank in:

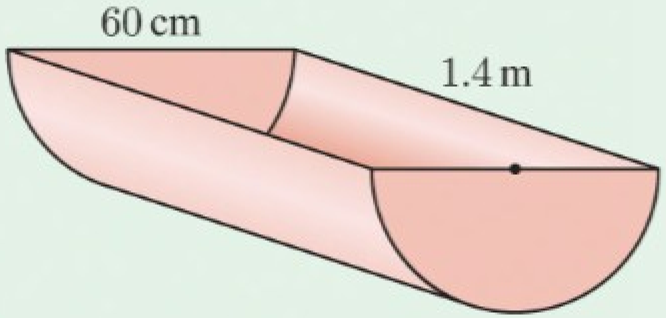
a cm^3 **b** m^3.

11 Find the capacity of:

a



b



12 A rectangular shed has a roof of length 12 m and width 5.5 m. Rainfall from the roof runs into a cylindrical tank with base diameter 4.35 m. If 15.4 mm of rain falls, how many millimetres does the water level in the tank rise?

13 A feed silo is made out of sheet steel using a hemisphere, a cylinder, and a cone.

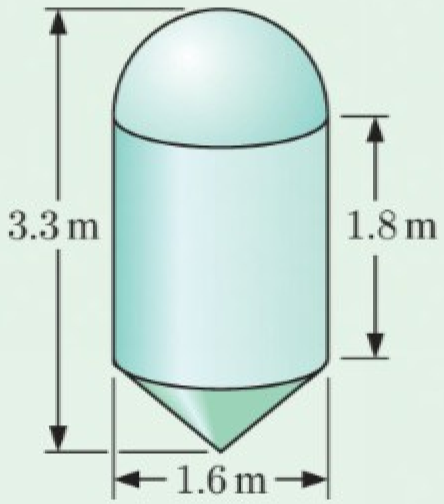
a Explain why the height of the cone must be 70 cm.

b Hence find the *slant height* of the conical section.

c Calculate the total amount of steel used.

d Show that the silo can hold about 5.2 cubic metres of grain.

e Write the capacity of the silo in kL.



1 **a** ≈ 18.3 cm **b** ≈ 38.3 cm **c** ≈ 91.6 cm^2

2 ≈ 10.4 cm

3 **a** ≈ 377.0 cm^2 **b** ≈ 339.8 cm^2 **c** ≈ 201.1 cm^2

4 **a** 71 m^2 **b** \$239.25 **5** ≈ 2670 m^2

6 **a** ≈ 4.99 m^3 **b** 853 cm^3 **c** ≈ 0.452 m^3

7 ≈ 3.22 m^3 **8** 16 333 spikes **9** $\approx 82\,400$ cm^3

10 **a** 65 000 cm^3 **b** 0.065 m^3

11 **a** 734.44 mL **b** ≈ 198 L **12** ≈ 68.4 mm

13 **a** height $= 3.3$ m $- 1.8$ m $- 0.8$ m $= 0.7$ m $= 70$ cm

b ≈ 1.06 m **c** ≈ 15.7 m^2

d **Hint:** Volume of silo
= volume of hemisphere + volume of cylinder
+ volume of cone

e ≈ 5.2 kL